Date: 12.12.2013
Teacher: Gülhan Can
Number of Students: 12
Grade Level: 10
Time Frame: 80 minutes

Relationships Between Lines

1. Goal(s)
   - The students will understand relationships between lines in a plane.

2A. Specific Objectives (measurable)
   - For given a line equation, students will state its direction vector.
   - For given two equations of lines, students will discuss whether the direction vectors of the lines are linearly dependent or not.
   - For given two equations of lines, students will sketch the graphs of the lines on coordinate system.
   - For given parallel, coincident, and intersecting lines, students will compare the relationships between the lines and linearly dependent or independent direction vectors of the lines.
   - For given parallel, coincident, and intersecting lines, students will formulate the relationships between the lines and linearly dependent or independent direction vectors of the lines.
   - For given two equations of lines, students will decide whether the lines are parallel, coincident or intersecting according to the direction vectors of the lines.

2B. Ministry of National Education (MoNE) Objectives
   - İki doğrunun birbirine gore durumlarını yorumlar ve uygulamalar yapar.

2C. NCTM-CCSS-IB or IGCSE Standards:
   - Choose one of the other curricular frameworks and include standards/objectives appropriate to your lesson.

3. Rationale
   - The purpose of this lesson is to provide students the capability of spatial thinking. In this lesson, students will identify relationships between lines in a plane. So, it will prepare the ground for vision and thinking skills in 3-D.
   - The students will learn about the enormous contributions of astronomy to geometry in a historical perspective. The students will interpret

a lunar eclipse in terms of the position of lines.

- The students will appreciate the geometry surrounding them. For example people benefit from the knowledge of relationships between lines to construct roads, railways, buildings; to decide navigational routes and flight paths etc. Additionally in science lessons, for given data, students will be able to decide whether they indicate a linear equation; if it is, what the relationships and their meanings in real life are (e.g. the velocity and time graphs for two different cars). So, students will be able to link their learning to other subject areas such as science, technology and engineering.

4. Materials
    - For an exploration activity, the students will need 12 worksheets for *worksheet1*, 12 worksheets for *worksheet2*, and 12 worksheets for *worksheet3*.
    - A projector and a computer will be used for the PowerPoint presentation (grade10_relationships between lines).
5. Resources
    - Ortaöğretim Geometri 10 (MEB yayınları) (4. Ünite: Doğrular)
    - Mathematics for the international student Mathematics SL (Chapter 13: Vector Applications)
    - Mathematics for the international student Mathematics HL (Chapter 15: Vector Applications)
    - http://www.bilimvetarih.com/node/58 for the historical perspective of the lesson
6. Getting Ready for the Lesson (Preparation Information)
    - During the activity, worksheets must be distributed respectively. After finishing *worksheet1*, the second will follow it and then the third one (it will take approximately six minutes for each worksheet).
    - Each student will have his/her own worksheet, but the students will study in pairs with their own desk mates.
    - Additionally, a checklist will be used while monitoring the students during the activity.
    - A sample of the worksheets, the checklist, and the PowerPoint representation are placed at the end of this plan.
    - Prepare 12 pieces of paper for students. Ask the students to write their names on the papers and put it on the desk as you can see; explain you want it to call them with their names. Approaching the end of the class, ask the students to write a reflection of the class with a few sentences on these pieces of paper and gather them.
7. Prior Background Knowledge (Prerequisite Skills)
    - The students have already known about coordinate system, sketching a line graph on the coordinate system, vectors in the plane from their 9^{th} grade courses. Additionally the students are able to write a vector equation by using a direction vector and translate this equation

into other forms such as parametric equation and Cartesian equation. The students are able to identify linearly dependent and independent vectors.

- Students must have the capability of reasoning and proving to find out the rules or any other patterns for given equation systems. Also, the students must be able to use the mathematical language and terminology to communicate with each other during the pair work and to represent their findings to the rest of the class.

Lesson Procedures

Transition: I wonder your ideas about a question: Does geometry come from space?

8A. Engage (3 minutes)

- After raising the question "Does geometry come from space?", wait for a while. By the way, reflect the first page of the presentation on the board.
- Then say "I want to tell you something about it. In ancient times, you know, it was a very fertile land around the Nile. So, it was vital to know what time Nile River would overflow. Therefore, people observed celestial bodies in the sky to make sense of the time… This is one of the most known examples that tell us astronomy made enormous contributions to geometry. So, what do you think now, does geometry come from space?" "Actually, yes…I think so."

Transition: Now, let's look at this picture.

B. Explore (35 minutes)

- Reflect the second page of the presentation on the board and say "It illustrates a lunar eclipse. You see the Sun, Earth, Moon and the lines. What do you think about the position of these lines?" "Are there any relationships between these lines?" Give time the students to think. Most probably the students will quickly recognize the intersecting lines. Take responses of the students and reflect the third page of the presentation.
- Say "Now, let's have a look to these railways. If you think the railways as a line, then can you observe any relationships between these lines?" Some of the lines are parallel, some of them are intersecting. Students most probably will tell and show them. If nobody says about coincident lines, express it with such an example "What if one of the railways is broken and need to be repaired? A new railway with the same properties of the old one will put there in the same direction, right? So, these old and new railways with the same properties are an example of coincident lines."
- Draw the intersecting, parallel, and coincident lines basically on the board and label each of them. Tell them it is easy to see the relations between lines in such a way but what if there are just equations of lines? Could we determine the relationships without sketching any graphs? Say them "Please keep it in your minds. Now, we will do an activity but first I want to remind you just a few things that will be

useful for the activity."

- There will be a small revision part. Take a board marker and write *ax + by + c=0* on the board and ask what it indicates. Take the response that it is a line equation. Then write *x + y = 1* and find its x-intercept and y-intercept by explaining briefly, then sketch it.
- Secondly with a new bullet, remind that direction vector lies in the same direction with its line. Look back to the graph on the board, show the direction vector can be both $\vec{u}$ (1, -1) or $\vec{v}$ (-1, 1). So, in general, we can express the direction vector as *(b, -a)* or *(-b, a).*
- Lastly remind the students, in a plane, if we can write a vector as a real number multiple of another vector, then we can say these vectors are linearly dependent; otherwise linearly independent.
- After the revision part, explain the steps of the activity: "First of all, I want you to work in pairs with your desk mates. Each of you will have same worksheets to take your own notes, but discuss your findings and study together during the activity." Show the first worksheet and say "I will give you a worksheet with three exercises on it. For each exercise, there are two equations of lines. Find out the direction vectors of the lines and sketch the graphs by using x-intercept and y-intercept." "After finishing all, consider if there is anything noticeable? A pattern, rule etc. If there is, record it at the end of the page with a sentence briefly." "You have six minutes after I distribute the worksheets."
- Distribute worksheets and use the checklist while monitoring the students. After six minutes distribute the second worksheets and say they have five minutes this time. Similarly follow the same steps and the students will have four minutes for the last worksheet.

Transition: Now, OK. then. Let's talk about our findings.

C. Explain (20 minutes)

- Say "First group, please present your outcomes shortly." Take the responses of each group and want them to justify their findings. After that, say "OK class. You all well done! I am really impressed; you behaved such good mathematicians do! Let's gather all these together and write it."
- Allocate a part on the board for conclusion. Use the last three pages of the presentation, the worksheets are also placed there respectively, you can benefit them while explaining the outcomes.

  Conclusion:

  1- If direction vectors are linearly dependent; the lines are parallel or coincident. If direction vectors are linearly independent; the lines are intersecting.

  2- For d_1: $a_1x + b_1y + c_1 = 0$ and d_2: $a_2x + b_2y + c_2 = 0$,

  If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the lines are coincident; if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the lines are parallel.

3- If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the lines are intersecting. For intersecting lines there is an intersecting point. Show it by using one of the exercises on worksheet3, sketch the graph on the board and find out the intersecting point. Then explain that students can find out the point by solving the line equations simultaneously. Solve the equation system and show that you find out the same point again.

Transition: You all do well and I have one more thing for you.

D. Extend (15 minutes)

- Tell that the students have already known about the relationships between lines. Ask them to consider whether it can be adopted to any real life situations? Encourage them to think about the uses of the line equations in science, engineering etc. and want them to model the situation, explain and justify it according to his/her learning. Give a clue, for example ask "What do you think an engineer considers constructing a building? How does he/she decide the positions of walls, corridors etc.? If you were an engineer what would you do then?"
- Give time to construct and present their ideas.

Transition: Thank you for this nice lesson, class. Lastly, I want you to write a few sentences as a reflection of this class.

E. Evaluate (3 minutes)

- The checklist has already used during the activity while monitoring students individually and as a group.
- At the end of the lesson, want them to write a few sentences on the papers that you have already distributed it to write their names on at the beginning of the lesson. Explain what you expect them to write: "Thank you for this nice lesson, class. Lastly, I want you to write a few sentences as a reflection of this class. For example, what was the most interesting mathematical idea in this class according to you? Or, have you learned/liked something new in this class. Please feel free if you want to add any other things about today's class."

9. Closure & Relevance for Future Learning

- Summarize the class briefly. Say "So, we find out about intersecting lines and you see there is an angle between these lines. But how can we learn about these angles and what are the uses of them? Have you ever heard about *slope*? Then, you will wait for it until the next lesson."

10. Specific Key Questions:

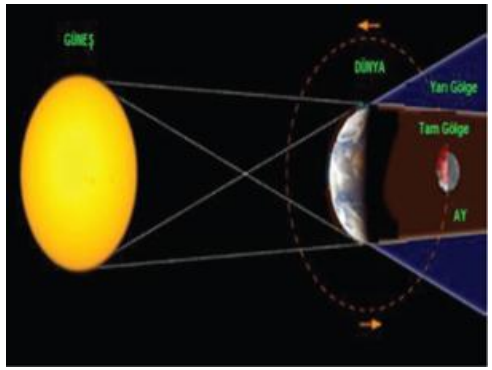
- What does $ax + by + c = 0$ define? (Knowledge)
- What is a *direction vector* of a line? (Knowledge)
- Can you explain the meaning of *linearly dependent* and *independent*? (Comprehension)
- In a plane, what can be the relationships between two lines? (Comprehension)

- For given a line equation, can you sketch the graph? How? (Application)
- What are your interpretations about the connections between the coefficients of the lines and the relationships of the lines? (Application)
- Can you compare linearly dependent and independent vectors? (Analysis)
- Have you got any rules, patterns for the given lines? Can you formulate them? (Synthesis)
- Can you construct any other life-related examples according to your learning? (Synthesis)
- How do the direction vectors affect the relationships between lines? (Evaluation)
- What is your justification that in worksheet1, you recorded the lines are parallel? (Evaluation)
- For given two line equations in a plane, can you decide the relationships between the lines without sketching any graphs? (Evaluation)

11. Modifications
    - During the lesson, especially in the revision part, if students have difficulty to remember their prior knowledge, extend the revision part with much more exercises and real life problems, and plan one more lesson time to do the exploration activity.
    - If some students do well, use extended problem which is stated in the extension part. Or, if you need one more, you can ask "For intersecting lines, how can you find out the angle between the lines?" and want them to consider on it in pairs. The students can have their own approaches by using their prior knowledge. For example some of them may choose two lines, sketch them on the coordinate system and try to estimate the angle. By doing it a few times, they may try to generalize their findings. Maybe some of them can use cosine theorem etc. While they were investigating, ask leading questions to them and want them to record their findings. Remember to use praises, appreciate, and encourage them.
    - For struggling students, pay attention to be a more successful student academically to help them in the group work.

Grade 10_relationships between lines (the first three pages are placed below; worksheet1, 2, and 3 are respectively placed on the last three pages)

Geometri Gökten Mi Düştü?



İki Doğrunun Birbirine Göre Durumları

Çalışma 1

Aşağıda verilen doğru denklemlerine ait doğrultman vektörlerini bularak, doğru denklemlerinin grafiğini çiziniz.

1- d_1: $2x + y + 4 = 0$

d_2: $-6x - 3y + 9 = 0$

2- d_3: $-\frac{x}{5} + 2y - 1 = 0$

d_4: $-x + 10y + 5 = 0$

3- d_5: $2\sqrt{2}\,x - 3y - \sqrt{32} = 0$

d_6: $-\frac{3}{2}y + \sqrt{2}\,x + \sqrt{18} = 0$

Bu üç örneği ve bulgularınızı tekrar inceleyiniz. Herhangi bir kural, örüntü vs. olduğunu düşünüyorsanız, kısaca kaydediniz:

Çalışma 2

Aşağıda verilen doğru denklemlerine ait doğrultman vektörlerini bularak, doğru denklemlerinin grafiğini çiziniz.

1- d_1: $2x + y + 4 = 0$

d_2: $-6x - 3y - 12 = 0$

2- d_3: $-\frac{x}{5} + 2y - 1 = 0$

d_4: $-x + 10y - 5 = 0$

3- d_5: $2\sqrt{2}\,x - 3y - \sqrt{32} = 0$

d_6: $-\frac{3}{2}y + \sqrt{2}\,x + \sqrt{8} = 0$

Bu üç örneği ve bulgularınızı tekrar inceleyiniz. Herhangi bir kural, örüntü vs. olduğunu düşünüyorsanız, kısaca kaydediniz:

Çalışma 3

Aşağıda verilen doğru denklemlerine ait doğrultman vektörlerini bularak, doğru denklemlerinin grafiğini çiziniz.

1- d_1: $2x + y + 4 = 0$
d_2: $x - y + 5 = 0$

2- d_3: $-\frac{x}{5} + 2y - 1 = 0$
d_4: $x + y - 6 = 0$

3- d_5: $2\sqrt{2}\,x - 3y - \sqrt{32} = 0$
d_6: $-2\sqrt{2}\,x + 4y + \sqrt{18} = 0$

Bu üç örneği ve bulgularınızı tekrar inceleyiniz. Herhangi bir kural, örüntü vs. olduğunu düşünüyorsanız, kısaca kaydediniz:

Grade10_Checklist_Relationships Between Lines

Groups	Names	Direction vectors			Linearly dependent/ind.			Pattern		
		W1	W2	W3	W1	W2	W3	W1	W2	W3
G1										
G1										
G2										
G2										
G3										
G3										
G4										
G4										
G5										
G5										
G6										
G6										