Date: 24.03.2014
Teacher: Gülhan Can
Number of Students: 19
Grade Level: 11
Time Frame: 45 minutes

Σ NOTATION

1. Goal(s)
- The students will understand the rationale of using Σ notation.

2A. Specific Objectives (measurable)
- Students will apply properties of Σ notation for given exercises.
- For given expanded form(s), students will rewrite the expression(s) by using Σ notation.
- Students will calculate the summations of given expressions by utilizing from formulae of Σ notation.

2B. Ministry of National Education (MoNE) Objectives
- Toplam sembolünü açıklar, kullanışları ile ilgili özellikleri açıklar ve temel toplam formüllerini modelleyerek inşa eder.

2C. NCTM-CCSS-IB or IGCSE Standards:
- Students will be able to expand a sum expressed in Σ notation. (IB SL)
- Students will be able to write an expanded expression in Σ notation. (IB SL)

3. Rationale
- The purpose of this lesson is to provide students the capability of realizing any pattern/ rules and constructing their own formulas/approaches/algorithms. In this lesson, it will be prepared a ground for the following topic *sequences and series*.
- It includes also solid examples of induction method to get the formulas and how these formulas make the problems easier.

4. Materials
- A projector and a computer (or smart board) will be used for the PowerPoint presentation (grade11_ Σ Notation) in the process of conducting the lesson.

5. Resources
- Ortaöğretim Matematik 11. Sınıf Ders Kitabı (MEB yayınları) (4. Ünite: Tümevarım ve Diziler)
- Mathematics for the international student Mathematics SL (Chapter 6: Sequences and Series)

- Mathematics for the international student Mathematics HL (Chapter 7: Sequences and Series)
- Algebra and Trigonometry Structure and Method Book 2 (by McDougal Littell) (11-Sequences and Series)

6. Getting Ready for the Lesson (Preparation Information)

- Before the lesson (in the previous class) talk to the students for a few minutes and mention them the activity you planned to conduct for the following class. Remind them they would work in groups of four or five for the activity and one of them would be the agent/spokesman of the group. So, to prevent a chaotic environment and to save time from the beginning of the activity, they must be already decided on their groups.
- Remind students to take their TI calculators into the class. They would be allowed to use the calculator for several questions.
- The second important thing is be ready in the classroom before the class to arrange the seats into clusters for the group work. While students are entering the class, remind them to sit with their own group-quickly.
- It should be useful to use slides while conducting the lesson. All the exercises and questions are already on it. However there is the hard copy of the questions at the end of the plan.
- A checklist will be used while monitoring the students during the activity.
- The checklist and the presentation are placed at the end of this plan.

7. Prior Background Knowledge (Prerequisite Skills)

- Students must have the capability of reasoning and proving to find out the rules or any other patterns for given summation. Also, the students must be able to use the mathematical language and terminology to communicate with each other during the pair work and to represent their findings to the rest of the class.
- Students have already learned about mathematical induction. Additionally, they have the prior basic knowledge on complex numbers, logarithm, trigonometry and functions to be able to solve miscellaneous questions.

Lesson Procedures

Transition: Have you ever heard about the legend about chess and wheat?

8A. Engage (3 minutes)

- Talk about the opening problem given below:

OPENING PROBLEM — THE LEGEND OF SISSA IBN DAHIR

Around 1260 AD, the Kurdish historian Ibn Khallikān recorded the following story about Sissa ibn Dahir and a chess game against the Indian King Shihram. (The story is also told in the Legend of the Ambalappuzha Paal Payasam, where the Lord Krishna takes the place of Sissa ibn Dahir, and they play a game of chess with the prize of rice grains rather than wheat.)

King Shihram was a tyrant king, and his subject Sissa ibn Dahir wanted to teach him how important all of his people were. He invented the game of chess for the king, and the king was greatly impressed. He insisted on Sissa ibn Dahir naming his reward, and the wise man asked for one grain of wheat for the first square, two grains of wheat for the second square, four grains of wheat for the third square, and so on, doubling the wheat on each successive square on the board.

The king laughed at first and agreed, for there was so little grain on the first few squares. By halfway he was surprised at the amount of grain being paid, and soon he realised his great error: that he owed more grain than there was in the world.

Things to think about:

a How can we describe the number of grains of wheat for each square?

b What expression gives the number of grains of wheat for the nth square?

c Find the total number of grains of wheat that the king owed.

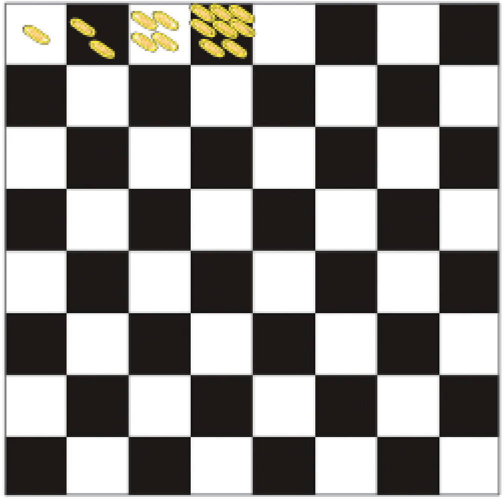
- Tell the legend shortly with a few sentences and be sure that the students understand the pattern that the Brahman conducted.
- Pay attention to the three questions given. You might use them to lead students during the activity.

Transition: Now, here is the question: Why do you think the king felt nervous in time? How many grains of wheat that the king owed?

B. Explore (5+5=10 minutes)

- As you told the students before, now explain the students about group work shortly and clearly before distribute a sheet to each group: “You will work with your group for five minutes to come up with an idea/method or an approach. If possible find out an approximate/exact value. After five minutes we will listen to an agent from each group”. The students are allowed to use calculator for the activity. Also remind them to record their work on the sheets you distributed.

- During these five minutes, be aware of the time; monitor the students with a checklist. Pay attention to *accountability* and *withitness*.
- Encourage students to make predictions, develop their own methods/approaches, and get their attention by asking where and how to use their method.
- After five minutes, let the group agents to present their ideas with a few sentences. Gather the different approaches (if there is) and use the chess table given below to model the question and come up with the pattern and without giving the formula, if necessary give the TI84 calculator instructions how to calculate such a summation.



- After the activity, ask students “Where do you think you can use your methods?” “Why it is useful to formulate such problems?” By asking such questions, lead them to be able to construct the concept of sequences and series. Do not give the details of sequences or series; but provide them a conceptual understanding of using notations such as Σ.

Transition: Now, OK. then. Here is the secret to be able to calculate such summations: Σ notation.

C. Explain (25 minutes)

- Explain that Σ notation is used to express the sum in series. (If necessary, emphasize the difference between sequences and series)
- Identify the *index, limits, summand* and reading of the expressions with Σ.
- Construct the properties of Σ by giving basic examples and basic proofs. (Properties will be written on the smart board but make them

clear on the next board by your examples)

- Similarly, give the formulas and remind them they have already learned about mathematical induction so you will not do it once more.
- Solve the examples about applications of Σ notation. (Use properties and formulas)

Transition: You all do well and I have one more thing for you. It is a nice question from LYS 2010 Exam.

D. Extend (5 minutes)

- For the last question they were asked, the students should realize to use their *modular arithmetic* knowledge in addition to the current topic requirements. Be sure all the students understand what the question asks by asking leading questions such as:
- "What must we divide by 5?" "How can we calculate the summation?" "Is it necessary to find out the whole number?" "Could you think a shorter way to find out the remainder without calculating the whole number?"
- Support students to analyze the question and develop their own approach by making meaningful connections. They can discuss their opinions with their group mates.
- Give time to construct and present their ideas.

Transition: Thank you for this nice lesson, class. Lastly, I want you to write a few sentences as a reflection of this class.

E. Evaluate (2 minutes)

- The checklist has already used during the activity while monitoring students individually and as a group.
- At the end of the lesson, want them to write a few sentences on the papers that you have already distributed it to write their findings at the beginning of the lesson. Explain what you expect them to write: "Thank you for this nice lesson, class. Lastly, I want you to write a few sentences as a reflection of this class. For example, what was the most interesting mathematical idea in this class according to you? Or, have you learned/liked something new in this class. Please feel free if you want to add any other things about today's class."

9. Closure & Relevance for Future Learning

- Summarize the class briefly. Remind the importance of this class for the following topic *sequences and series.*

10. Specific Key Questions:

- What must we divide by 5? (Knowledge)
- How can we calculate the summation? (Application)

- Is it necessary to find out the whole number? (Analysis)
- Could you think a shorter way to find out the remainder without calculating the whole number? (Analysis)
- Where do you think you can use your methods? (Synthesis)
- What does Σ notation provide us? (Evaluation)
- Why it is useful to formulate such problems? (Evaluation)

11. Modifications
    - If some students do well, use extended problem which is stated in the extension part. While they were investigating, ask leading questions to them and want them to record their findings. Remember to use praises, appreciate, and encourage them.
    - For struggling students, pay attention: An academically successful student(s) could help them in the group work.

Questions:

1- $f(x) = 2x + 3$ and $x_n = 3n + 1$,

Then, what is the value of the summation $\sum_{k=1}^{3} x_k f(x_k)$?

2- $\sum_{k=1}^{5}\sum_{r=1}^{5}(k.r) = ?$

3- $\sum_{k=1}^{63}\log_2\left(1+\frac{1}{k}\right) = ?$

4- $\sum_{k=1}^{99}\left(\sqrt{k+1}-\sqrt{k}\right) = ?$

5- $\sum_{k=1}^{90}\sin^2 k = ?$

6- $\sum_{k=1}^{20}(-1)^k(5k+1) = ?$

7- $\sum_{k=-5}^{5}\left(k^3+k^2+k\right) = ?$

8- $\sum_{k=1}^{10}(3k-4) = ?$

9- $\sum_{k=1}^{15}k^2 = ?$

10- $\sum_{k=1}^{10}k(k-4) = ?$

11- $\sum_{k=5}^{10}k^3 = ?$

REVIEW QUESTIONS (Σ NOTATION)

1- $\sum_{k=5}^{12}\frac{1}{k^2-7k+12} = ?$

2- What is the reminder when the summation of $\sum_{k=0}^{20} k!$ is divided by 15?

3- $\sum_{k=1}^{10} e^{2\ln k} = ?$

4- $1+\frac{1}{1+2}+\frac{1}{1+2+3}+................+\frac{1}{1+2+3+..........+25} = ?$

5- $\sum_{k=-10}^{10} \frac{1+k+k^2+k^3+........+k^9}{1+k+k^2+k^3+k^4} = ?$

6- Extension: **(2010LYS)** What is the reminder when the summation of $\sum_{n=0}^{100} 3^n$ is divided by 5?

Extension:

a Expand $\sum_{k=1}^{n} k$.

b Now write the sum with terms in the reverse order, placing each term under a term in the original expansion. Add each term with the one under it.

c Hence write an expression for the sum S_n of the first n integers.

d Hence find a and b if $\sum_{k=1}^{n} (ak+b) = 8n^2 + 11n$ for all positive integers n.

ACI_L3D_VERSION2 (for explanation part)

Explanation:

- Give the definition of Σ notation. Identify the *summand*, *index*, and *limits* of the summation.
- Give examples about how to expand the expressions in Σ notation such as:

$$\sum_{k=1}^{4} k^2 = 1^2 + 2^2 + 3^2 + 4^2$$

- Construct the properties of Σ notation by giving basic examples and proofs:

Example (explanations are written on the board)	Property (written on the smart board, after generalization)
$\sum_{k=1}^{4} 5 = 5+5+5+5 = 20$	$\sum_{k=1}^{n} c = n.c$ (c: constant)
$\sum_{k=1}^{5} 2k = 2.(1+2+3+4+5)$	$\sum_{k=1}^{n} c.a_k = c\sum_{k=1}^{n} a_k$
$\sum_{k=1}^{5} (k+5) = (1+2+3+4+5)+(5+5+5+5+5)$	$\sum_{k=1}^{n} (a_k \mp b_k) = \sum_{k=1}^{n} a_k \mp \sum_{k=1}^{n} b_k$
$\sum_{k=1}^{4} k^2 = (1^2+2^2+3^2)+(4^2+5^2+6^2+7^2+8^2)$	$\sum_{k=1}^{n} a_k = \sum_{k=1}^{p} a_k + \sum_{k=p+1}^{n} a_k$ (1<p<n)

- Formulas of Σ notation will be written on smart board.
- Exercises will be solved by using properties and formulas of Σ notation.

Exercises:

1- $\sum_{k=1}^{n-3} k = 21$ is given. Then, what is *n*?

2- f(k)=2k+3 and x_n=3n+1 are given. Then, $\sum_{k=1}^{3} x_k . f(k) = ?$

3- $\sum_{k=1}^{20} (-1)^k .(5k+1) = ?$

4- $\sum_{k=-5}^{5} k = ?$

5- $\sum_{k=-5}^{5} k^3 = ?$

6- $\sum_{k=1}^{10} k.(k-4) = ?$

7- $\sum_{k=1}^{10} (k+1).(k-1) = ?$

8- $\sum_{k=1}^{10} (k+1).(k-4) = ?$

9- $\sum_{k=1}^{180} \cos k^0 = ?$

10- $\sum_{k=1}^{90} \sin^2 k^0 = ?$

11- $\sum_{k=1}^{99} (\sqrt{k+1} - \sqrt{k}) = ?$

12- f(x)=3x+2 is given. $\sum_{k=1}^{50} (f(k+1) - f(k)) = ?$

13- $\sum_{k=1}^{63} \log_2 (1 + \frac{1}{k}) = ?$

14- $\sum_{k=5}^{10} k^3 = ?$

- At the end of the class (about five minutes), do the activity part. Give the instructions to the students. The students will choose any of the 19 cards. There will be some drill exercises written on the cards. Each student will have three minutes to calculate his/her own exercise. After the calculation, they will decide which letter matches with his/her answer. At the end, the conclusion box will be occurred which is:

L	I	S	E	3	D		M	A	T	H	E	M	A	T	I	C	S	!

D
100/101

L
91

H
120

M
69

A
232

E
-340

T
21


435

C
306

I
10

3
462

S
100

!
126

Worksheet_ Σ NOTATION

1- $\sum_{k=1}^{20} k =$
2- $\sum_{k=1}^{16} 3k - 5 =$
3- $\sum_{k=1}^{15} k^2 =$
4- $\sum_{k=-5}^{5} \left(k^3 + k^2 + k\right) =$

5- $\sum_{k=-4}^{5}\left(k^3-3k\right)=$

6- $\sum_{k=1}^{10}\frac{1}{k\left(k+1\right)}$

7- $\sum_{k=1}^{25}\left(-1\right)^k\left(3k+2\right)=$

8- $i^2=-1$ is known. Hence, $\sum_{k=1}^{50} i^k$ =?

9- $\sum_{k=1}^{10} e^{2\ln k} =$
10- $\sum_{k=1}^{6} \binom{6}{k} =$
11- f ve $g : N^+ \rightarrow N^+$, If $f(x) = \sum_{k=1}^{x} (k+1)$ and $g(x) = \sum_{k=1}^{x} 2k$ then, $f \circ g(4) = ?$

12- $\sum_{k=1}^{30} a_k = 123$, $\sum_{k=10}^{30} a_k = 51$ and $\sum_{k=1}^{20} a_k = 72$ are given.

Then, $\sum_{k=10}^{20} a_k$ =?

13- Express $1+4+7+10+........+130$ in sigma notation and evaluate.

14- Express $7+11+15+........+87$ in sigma notation and evaluate.

15- Express $3.2+5.5+7.8+..........+41.59$ in sigma notation and evaluate.