Date: 06.03.2014
Teacher: Gülhan Can
Number of Students: 20
Grade Level: 11
Time Frame: 45 minutes

Σ and π NOTATION

1. Goal(s)
- The students will understand where and how to use the notations Σ and π.

2A. Specific Objectives (measurable)
- For a given summation, students will realize appropriate summation formula and apply it on the summation.
- Students will apply the properties of Σ notation such as *changing the boards* by adding or subtracting appropriate terms.
- Students will construct the appropriate formula for an expanded expression.
- Students will synthesis their prior knowledge with the Σ and π notation.

2B. Ministry of National Education (MoNE) Objectives
- Toplam sembolünü ve çarpım sembolünü açıklar, kullanışları ile ilgili özellikleri açıklar ve temel toplam formüllerini modelleyerek inşa eder.

2C. NCTM-CCSS-IB or IGCSE Standards:
- Choose one of the other curricular frameworks and include standards/objectives appropriate to your lesson.

3. Rationale
- The purpose of this lesson is to provide students the capability of realizing any pattern/ rules and constructing their own formulas/approaches/algorithms. In this lesson, it will be prepared a ground for the following topic *sequences and series*.
- It includes also solid examples of induction method to get the formulas and how these formulas make the problems easier.

4. Materials
- A projector and a computer will be used for the PowerPoint presentation (grade11_ Σ and π Notation).

5. Resources
- Ortaöğretim Matematik 11. Sınıf Ders Kitabı (MEB yayınları) (4. Ünite: Tümevarım ve Diziler)
- Mathematics for the international student Mathematics SL (Chapter 6: Sequences and Series)

- Mathematics for the international student Mathematics HL (Chapter 7: Sequences and Series)

6. Getting Ready for the Lesson (Preparation Information)

- It should be useful to use slides while conducting the lesson. All the review questions are already on it. However there is the hard copy of the questions at the end of the plan.
- The students are allowed to work in pairs with their own desk mates.
- A checklist will be used while monitoring the students during the activity.
- The checklist and the PowerPoint presentation are placed at the end of this plan.
- Prepare 20 pieces of paper for students. Ask the students to write their names on the papers and put it on the desk as you can see; explain you want it to call them with their names. Approaching the end of the class, ask the students to write a reflection of the class with a few sentences on these pieces of paper and gather them.

7. Prior Background Knowledge (Prerequisite Skills)

- The students have already known about the formulas and priorities of Σ and π notation. Additionally they have the prior basic knowledge on complex numbers, logarithm, trigonometry and functions.
- Students must have the capability of reasoning and proving to find out the rules or any other patterns for given summation/product. Also, the students must be able to use the mathematical language and terminology to communicate with each other during the pair work and to represent their findings to the rest of the class.

Lesson Procedures

Transition: Have you ever heard about the relationship between chess and wheat?

8A. Engage (5 minutes)

- Talk about the opening problem given below:

OPENING PROBLEM **THE LEGEND OF SISSA IBN DAHIR**

Around 1260 AD, the Kurdish historian Ibn Khallikān recorded the following story about Sissa ibn Dahir and a chess game against the Indian King Shihram. (The story is also told in the Legend of the Ambalappuzha Paal Payasam, where the Lord Krishna takes the place of Sissa ibn Dahir, and they play a game of chess with the prize of rice grains rather than wheat.)

King Shihram was a tyrant king, and his subject Sissa ibn Dahir wanted to teach him how important all of his people were. He invented the game of chess for the king, and the king was greatly impressed. He insisted on Sissa ibn Dahir naming his reward, and the wise man asked for one grain of wheat for the first square, two grains of wheat for the second square, four grains of wheat for the third square, and so on, doubling the wheat on each successive square on the board.

The king laughed at first and agreed, for there was so little grain on the first few squares. By halfway he was surprised at the amount of grain being paid, and soon he realised his great error: that he owed more grain than there was in the world.

Things to think about:

a How can we describe the number of grains of wheat for each square?

b What expression gives the number of grains of wheat for the nth square?

c Find the total number of grains of wheat that the king owed.

- Pay attention to the three questions given. Ask each of them one by one. Give time students to think, after gathering the students' responses, move on the following question.
- Encourage students to make predictions, develop their own methods/approaches, and get their attention by asking where and how to use their method. (No calculations yet)

Transition: Now, let's have a closer look to the problem.

B. Explore (10 minutes)

- Ask students work in pairs (with their deskmates) and develop their method by using Σ notation and then, calculate for the result. Give them nearly five minutes to say their answers.

- After gathering the answers, if there is any misconception, you can ask students whether one of them would like to explain. Make clear all the points of the problem.
- Use the checklist while monitoring the students, take notes.
- After calculation, ask students "Where do you think you can use your methods?" "What does Σ notation provide us?" "Why it is useful to formulate such problems?" By asking such questions, lead them to be able to construct the concept of sequences and series. Do not give the details of sequences or series; but provide them a conceptual understanding of using notations such as Σ and π.

Transition: Now, OK. then. Let's talk about our findings.

C. Explain (20 minutes)

- Since students already learned about the topic and the class is much more a review class; use the miscellaneous questions which are both about Σ and π notation.
- There are 15 questions which of them are related with previous topics of the year such as logarithm and complex numbers. Additionally, some of the questions are related with the topics that students are already familiar-such as trigonometry, functions. Therefore, sometimes students need to be reminded. Give the key points for such situations.
- For each question give at least two minutes to students to consider on it. They are also allowed to discuss the questions with their deskmates. Ask for students' different solution techniques and be aware of teachable moments. If there is an original approach, then support the student(s) to represent the method he/she used to his/her peers.
- Use the time efficiently. There could be many questions for one session. So, you can skip some of them or you can use the rest of them for the next session.

Transition: You all do well and I have one more thing for you. It is a nice question from LYS 2010 Exam.

D. Extend (10 minutes)

- For the last question they were asked, the students should realize to use their *modular arithmetic* knowledge in addition to the current topic requirements. Be sure all the students understand what the question asks by asking leading questions such as:
- "What must we divide by 5?" "How can we calculate the summation?" "Is it necessary to find out the whole number?" "Could you think a shorter way to find out the remainder without calculating the whole number?"
- Support students to analyze the question and develop their own approach by making meaningful connections. They can discuss their opinions with their deskmates.
- Give time to construct and present their ideas.

Transition: Thank you for this nice lesson, class. Lastly, I want you to write a few sentences as a reflection of this class.

E. Evaluate (3 minutes)

- The checklist has already used during the activity while monitoring students individually and as a group.
- At the end of the lesson, want them to write a few sentences on the papers that you have already distributed it to write their names on at the beginning of the lesson. Explain what you expect them to write: "Thank you for this nice lesson, class. Lastly, I want you to write a few sentences as a reflection of this class. For example, what was the most interesting mathematical idea in this class according to you? Or, have you learned/liked something new in this class. Please feel free if you want to add any other things about today's class."

9. Closure & Relevance for Future Learning

- Summarize the class briefly. Remind the importance of this class for the following topic *sequences and series.*

10. Specific Key Questions:

- What must we divide by 5? (Knowledge)
- How can we calculate the summation? (Application)
- Is it necessary to find out the whole number? (Analysis)
- Could you think a shorter way to find out the remainder without calculating the whole number? (Analysis)
- Where do you think you can use your methods? (Synthesis)
- What does Σ notation provide us? (Evaluation)
- Why it is useful to formulate such problems? (Evaluation)

11. Modifications

- If some students do well, use extended problem which is stated in the extension part. While they were investigating, ask leading questions to them and want them to record their findings. Remember to use praises, appreciate, and encourage them.
- For struggling students, pay attention to be a more successful student academically to help them in the group work.

REVIEW QUESTIONS (Σ and π NOTATION)

1- $\sum_{k=5}^{12} \frac{1}{k^2-7k+12} = ?$

2- What is the reminder when the summation of $\sum_{k=0}^{20} k!$ is divided by 15?

3- $\sum_{k=1}^{10} e^{2\ln k} = ?$

4- $1+\frac{1}{1+2}+\frac{1}{1+2+3}+.....+\frac{1}{1+2+3+.....+25} = ?$

5- $\sum_{k=1}^{90} \sin^2 k = ?$

6- $\sum_{k=-10}^{10} \frac{1+k+k^2+k^3+.....+k^9}{1+k+k^2+k^3+k^4} = ?$

7- If $\prod_{k=1}^{n} 8^{\frac{1}{k(k+1)}} = \sqrt[3]{256}$ then what is the value of ***n***?

8- The number A is defined as $A = \prod_{k=1}^{8} 625.2^{k}$. Hence, how many digits does the number have?

9- $\prod_{k=2}^{63} \log_k (k+1) = ?$

10- $\prod_{k=1}^{15} \frac{k^2+3k+2}{k^2+4k} = ?$

11- x_1, x_2 are the roots of the equation $3x^2 - 4x - 2 = 0$. Hence, $\prod_{k=1}^{2} (3x_k - 1) = ?$

12- $f(x) = \sum_{k=1}^{x} k^2$ and

$g(x) = \prod_{k=2}^{x} \left(1 + \frac{1}{k}\right)$. Then what is the value of $f \circ g(11)$?

13- $\dfrac{\prod_{k=1}^{27}\left(k^2+k\right)}{\sum_{k=1}^{7}k} = ?$

14- $i^2 = -1;$

Then $\prod_{n=1}^{15}\dfrac{i^n}{\sqrt[5]{2}} = ?$

15- (2010LYS) What is the reminder when the summation of $\sum_{n=0}^{100}3^n$ is divided by 5?

Extension:

a Expand $\sum_{k=1}^{n}k$.

b Now write the sum with terms in the reverse order, placing each term under a term in the original expansion. Add each term with the one under it.

c Hence write an expression for the sum S_n of the first n integers.

d Hence find a and b if $\sum_{k=1}^{n}(ak+b) = 8n^2 + 11n$ for all positive integers n.