

Introduction:

The topic I chose is *Differentiation*. It is one of the challenging topics in secondary school, 12th grade level. In both MoNE and IB DP curricula, the topic begins with *introduction to differential calculus*. It means the derivative function and its use is explained by the *limit* and *rate of change* concept. Then the rules of differentiation, properties of curves, and applications of differential calculus place in the topic. It is a huge topic but since it includes many real-life related issues, I thought it would help me to construct different kinds of questions.

When I chose the questions, the first issue was learning outcomes-what I expect from the students. Here are the objectives that I intended to measure:

- 1- Students explain the concept of rate of change by utilizing from science and geometry models.
- 2- Students examine a function's differentiability at a point and at an interval.
- 3- Students explain the rules of differentiation and apply them in the questions.
- 4- Students construct the chain rule and calculate the derivative.
- 5- Students explain the second and higher order derivative.
- 6- For a given function, students find out the tangent and normal equations of the curve at a point.
- 7- Students identify the functions in which interval/when to increase and decrease according to the sign of the derivative.
- 8- Students examine the extremum points by using differentiation.
- 9- Students use applications of differentiation such as optimization problems, kinematics, and rate of change.

The second thing that I considered while designing questions was the possible mistakes students could do. I thought even what kind of mistakes I did in my past education and I utilized from my observations and experiences about which difficulties or misconceptions students might have. Therefore, especially in multiple choice questions, I constructed the alternatives according to this approach and tried to measure where the problematic issue occurs and what is the reason behind it.

Multiple Choice Questions:

1- $f(x) = \sqrt{2x+1}$ is given. What is the value of $f'(x)$ at the point $x=4$? [Application]

- A) 3
- B) $-9/2$
- C) $-1/3$
- D) $1/3$
- E) $1/6$

Answer: D

(Objective#2, #3)

2- What is the value of $f'(x)$ for $f(x) = 7x - \frac{4}{x} + \frac{3}{x^3}$? [Application]

- A) 6
- B) $7 + \frac{4}{x^2} - \frac{9}{x^4}$
- C) $3 + 9x^2$
- D) $7x - 4x^{-1} + 3x^{-3}$
- E) $7 + 4x^2 - 9x^4$

Answer: B

(Objective#3)

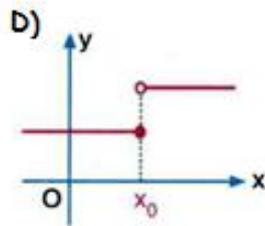
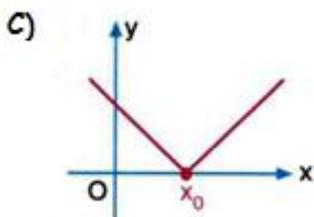
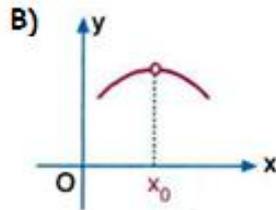
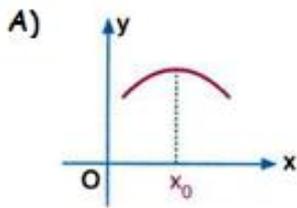
3- Which of the following represents the local maximum point of $f(x) = x^3 - 3x^2 - 9x + 5$? [Application]

- A) (3, -22)
- B) (-1, 15)
- C) (-1, 10)
- D) (-1, 0)
- E) (1, -12)

Answer: C

(Objective#3, #8)

- 4- For which of the following graphs of $y = f(x)$;
Does the function have a derivative at the point $x = x_0$? [Analysis]



E) None of them

Answer: A

(Objective#2)

- 5- $y = t^3 - t^2$,
 $t = 3u^2 - u + 1$,
 $u = 2x^2 + x + 1$ are given. What is $\frac{dy}{dx}$ at the point $x = 0$? [Application]

- A) 105
B) 0
C) 15
D) 5
E) 1

Answer: A

(Objective#4)

True/False Questions:

- 1- In any function, the gradient is constant for all values of x . (F) [Analysis] (Objective#3)
- 2- If we construct the derivative of $\tan x$ by utilizing from the knowledge $\tan x = \frac{\sin x}{\cos x}$; we use the *quotient rule*. (T) [Knowledge & Comprehension] (Objective#3)
- 3- The n^{th} derivative of y with respect to x is obtained by differentiating $y = f(x)$ n times. (T) [Knowledge & Comprehension] (Objective#5)
- 4- For a given curve, the gradient of the normal to the curve is the additional inverse of the gradient of the tangent to the same curve. (F) [Knowledge & Synthesis] (Objective#6)
- 5- Suppose D is an interval in the domain of $f(x)$; then $f(x)$ is increasing on $D \leftrightarrow f(a) \leq f(b)$ for all $a, b \in D$ such that $a < b$. (T) [Analysis] (Objective#7)

Fill in the blanks & Matching Questions:

- For the given $f(x)$ and $f'(x)$ functions; fill in the blanks and match the names of the rules for each function below (Assume c is a constant). [Knowledge & Application] (Objective#3)

$f(x)$	$f'(x)$
.....	0
x^n
$c.u(x)$
.....	$u'(x) + v'(x)$
$u(x).v(x)$

Name of the rule

- a) Addition rule
- b) Chain rule
- c) Product rule
- d) Constant times a function
- e) Differentiating any power of x
- f) Differentiating a constant

Answer:

$f(x)$	$f'(x)$	Answer	<u>Name of the rule</u>
c	0	f	a) Addition rule
x^n	$n \cdot x^{n-1}$	e	b) Chain rule
$c \cdot u(x)$	$c \cdot u'(x)$	d	c) Product rule
$u(x) + v(x)$	$u'(x) + v'(x)$	a	d) Constant times a function
$u(x) \cdot v(x)$	$u'(x) \cdot v(x) + v'(x) \cdot u(x)$	c	e) Differentiating any power of x
			f) Differentiating a constant

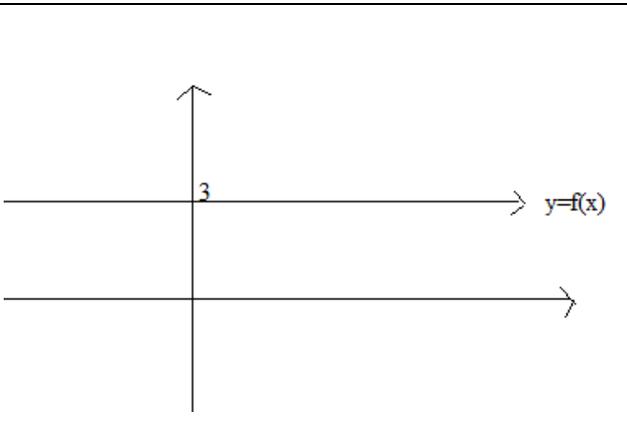
Short Answer Questions:

1- What is a gradient function of $y = f(x)$ called? [**Knowledge**]

Answer: Derivative function
(Objective#1)

2- What does the value of $f'(a)$ represent? [**Comprehension**]

Answer: the gradient of the tangent to $y = f(x)$ at the point where $x = a$.
(Objective#1)

3-		<ul style="list-style-type: none"> • What is $f(2) = ?$ [Application] • Answer: 3
		<ul style="list-style-type: none"> • What is $f'(2) = ?$ [Application] • Answer: 0

(Objective#3)

4- For a general function $y = f(x)$, where A is the point $(x, f(x))$ and B is the point $(x+h, f(x+h))$; find out (and write down) the gradient of chord [AB].

Answer:
$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

(Objective#1)

5- When do we call a function *strictly increasing* or *strictly decreasing*?

Answer:

- Strictly increasing: an increase in x produces an increase in y .
- Strictly decreasing: an increase in x produces a decrease in y .

(Objective#7)

Restricted Essay Question:

- 1- Interpret the reason of different signs in first derivation of a function. When does it have a positive sign and when does it have a negative sign? Explain the reason and show your interpretation by sketching a graph on the coordinate system. (max. 50 words) (**Objective#1, #7**)

Rubric:

	Needs to improve	Satisfactory	Excellent	Points
Explanation related with the topic	Explanation is not clear and not acceptable as a reason. There are missing parts in the prior background knowledge. (1-2)	The reasons are appropriate for the topic. Explanation is satisfactory. (3-5)	Explanation is clear and meaningful. Connections between prior background knowledge and the new topic are represented consistently. (6-8)	.../8
Interpreting by sketching a graph	There is no drawing on the coordinate system/ incorrect drawings of tangents/ incorrect explanation. (1-2)	Drawing tangent is appropriate but explanations have small gaps within the reason. (3-5)	Drawing is clear and tidy. Easy-understandable and correct interpretation. (6-8)	.../8
Organization and structure	Organization is confusing and disordered. Ideas and interpretation are not well-structured. (1-2)	Organization is well-designed and prepared clearly. (3-4)	Organization is rigorous and well-designed in general. Drawing and explanation are structured properly. (5-6)	.../6

Extended Essay Question:

- 1- How do you integrate your derivative knowledge to science, economics, or any other real life issues? Justify your ideas by giving at least one example and identify the usage of derivative in your example(s). **(Objective#9)**

Rubric:

	Needs to improve	Satisfactory	Excellent	Points
Explanation related with the given example	Given example is not real-life related and has not an acceptable explanation related with the topic. There are missing parts in the prior background knowledge. (1-2)	Given example is real-life related and appropriate for the topic. Explanation is satisfactory. (3-5)	Explanation about the given example is clear and meaningful. Connections between the knowledge of derivative and the given example are represented consistently. (6-8)	.../8
Justification of the given example by the use/properties of derivative.	There is no coherent justification of the example/ideas are represented poorly/ (1-2)	Justification about use of derivative (rate of change problems, gradient and tangent issues, kinematics, optimization...) is appropriate for the topic. (3-5)	More than one aspect/more than one (appropriate) example/ consistent justification/represents the ideas making connections with the topic/applies the theoretic knowledge into real-life/science issues. (6-8)	.../8
Organization and structure	Organization is confusing and disordered. Ideas and interpretation are not well-structured. (1-2)	Organization is well-designed and prepared clearly. (3-4)	Organization is rigorous and well-designed in general. Drawing and explanation are structured properly. (5-6)	.../6